HGMatch: A Match-by-Hyperedge Approach for Subgraph Matching on Hypergraphs

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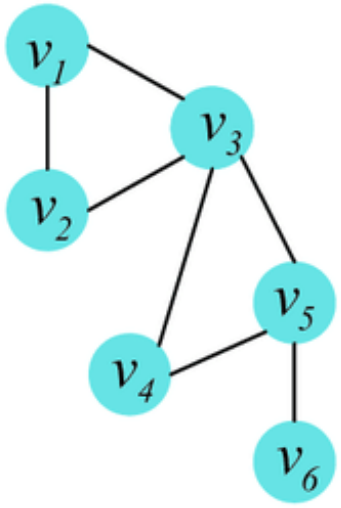
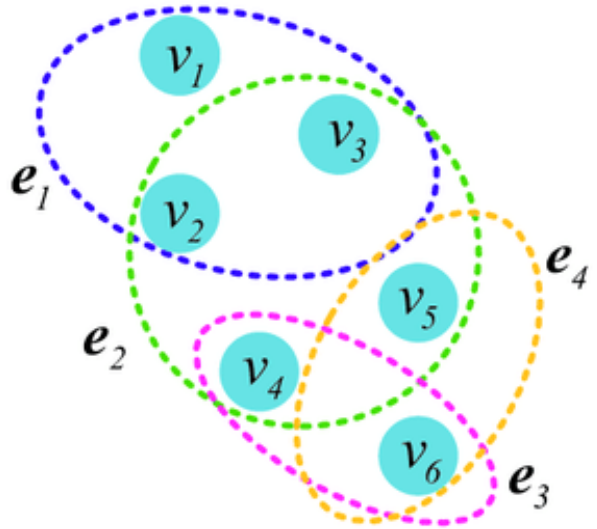
Graphs vs Hypergraphs

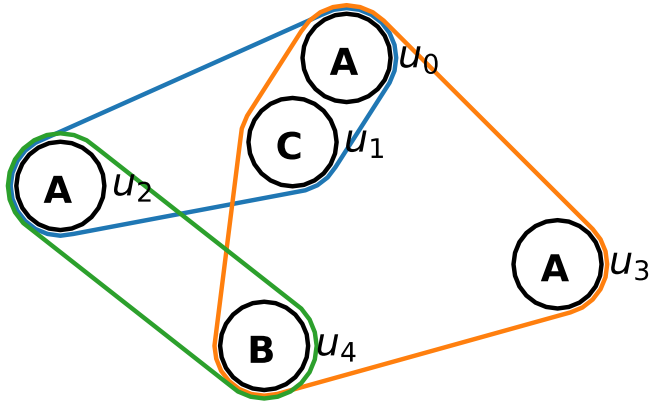


Graph

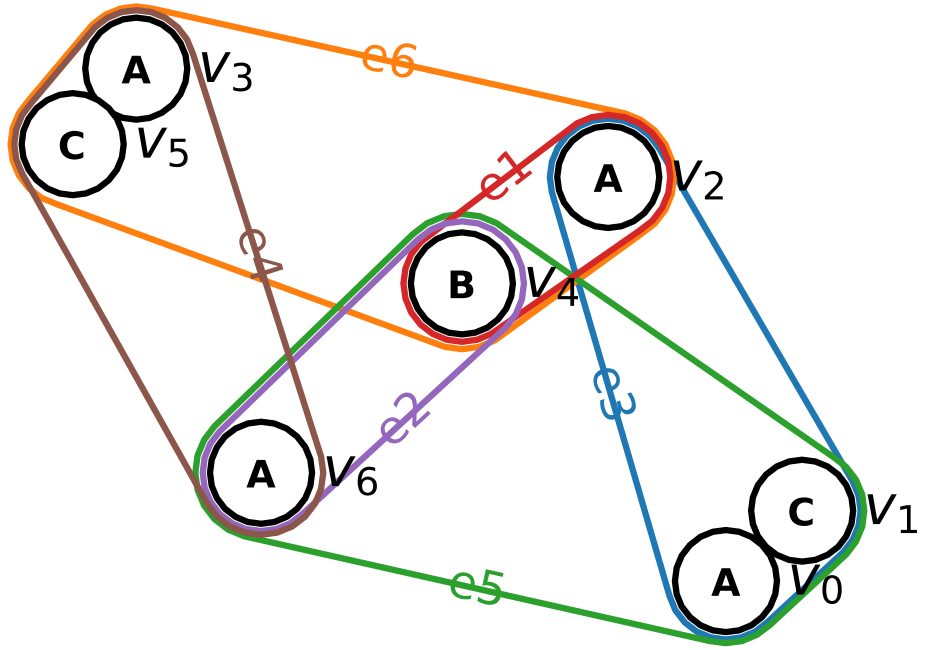


Hypergraph

Subgraph Matching on Hypergraphs

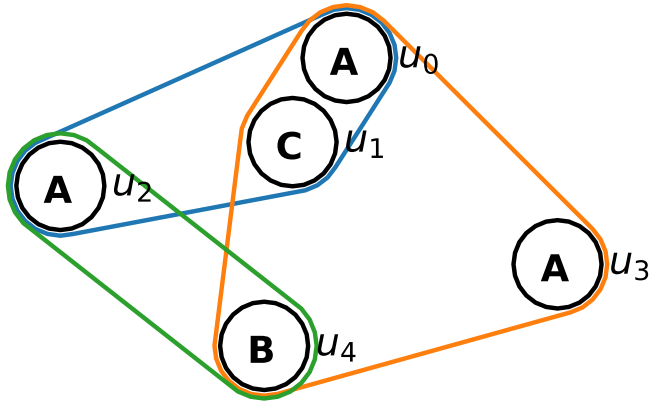


Query Hypergraph

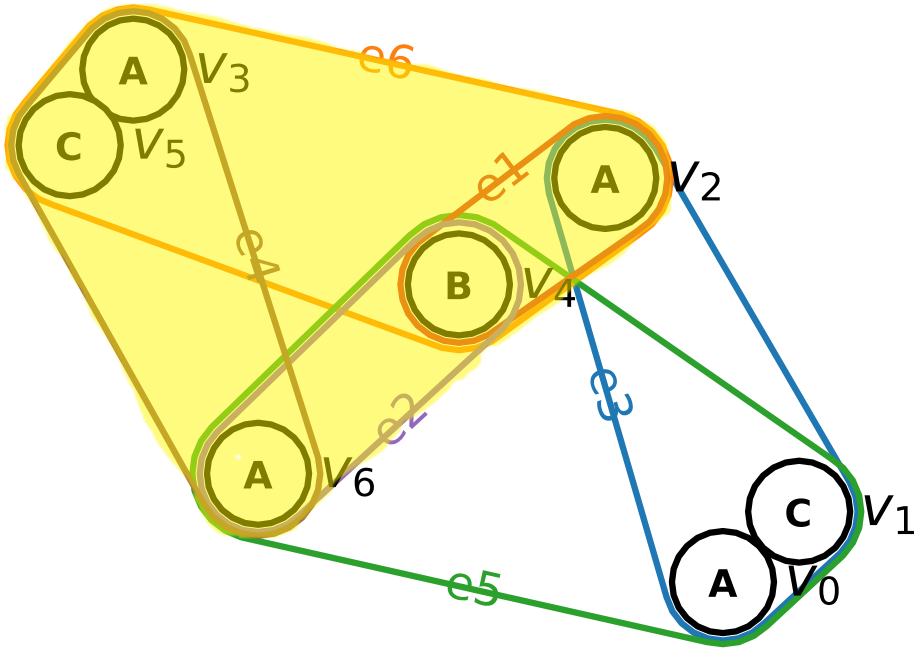


Data Hypergraph

Subgraph Matching on Hypergraphs

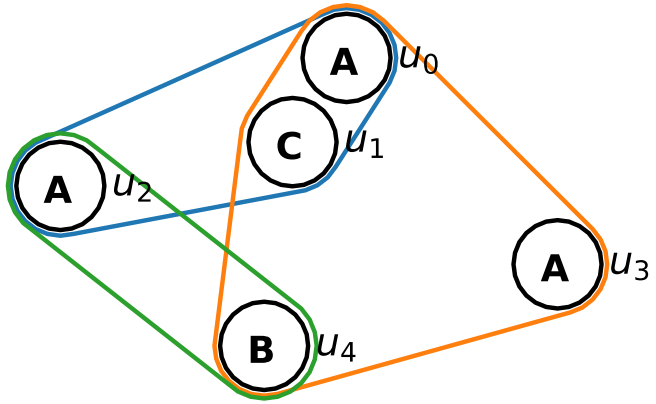


Query Hypergraph

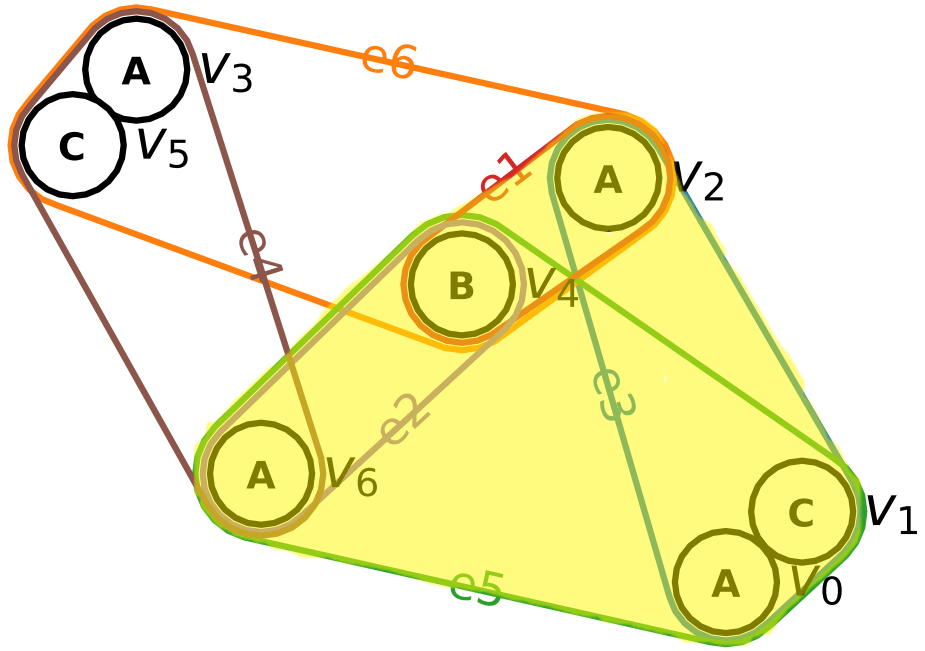


Data Hypergraph

Subgraph Matching on Hypergraphs



Query Hypergraph

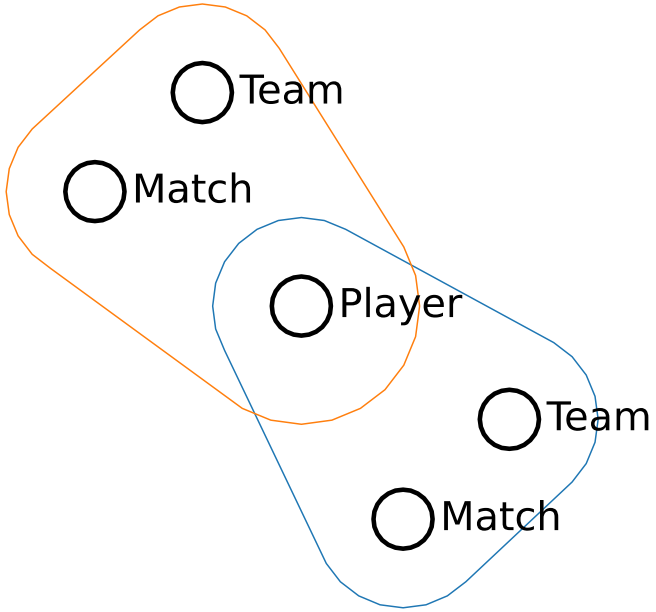


Data Hypergraph

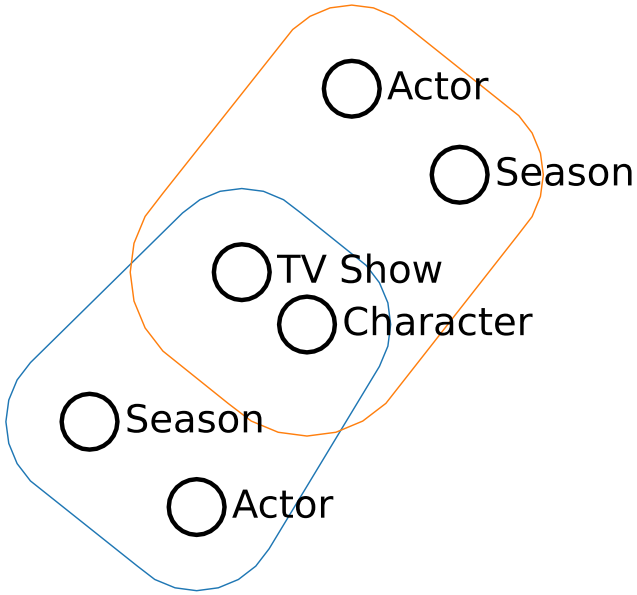
Applications

- **Mining Biological Networks**
  - e.g., protein interactions, gene interactions
- **Querying Hypergraph Databases**
  - e.g., AtomSpace, HyperGraphDB, TypeDB
- **Pattern Learning in NLP**
  - e.g., semantic hypergraphs (each word is a vertex, and each sentence is a hyperedge)
- **Q/A over Hypergraph Knowledge Base**
  - e.g., JF17K dataset (a subset of non-binary relations extracted from Freebase)

Example Queries for *JF17K* Dataset



Which football players represented different teams in different matches?



Which actors played the same character in a TV show on different seasons?

Strawman Approach

- Convert the hypergraph to a *bipartite graph* and apply existing *subgraph matching algorithms*
  - by taking the incidence matrix and treating this as the incidence matrix of a bipartite graph
- Directly extend existing subgraph matching *algorithms* to the case of *hypergraphs*
  - recursively expand the partial embedding vertex-by-vertex by mapping a query vertex to a data vertex following a given matching order and backtrack when necessary

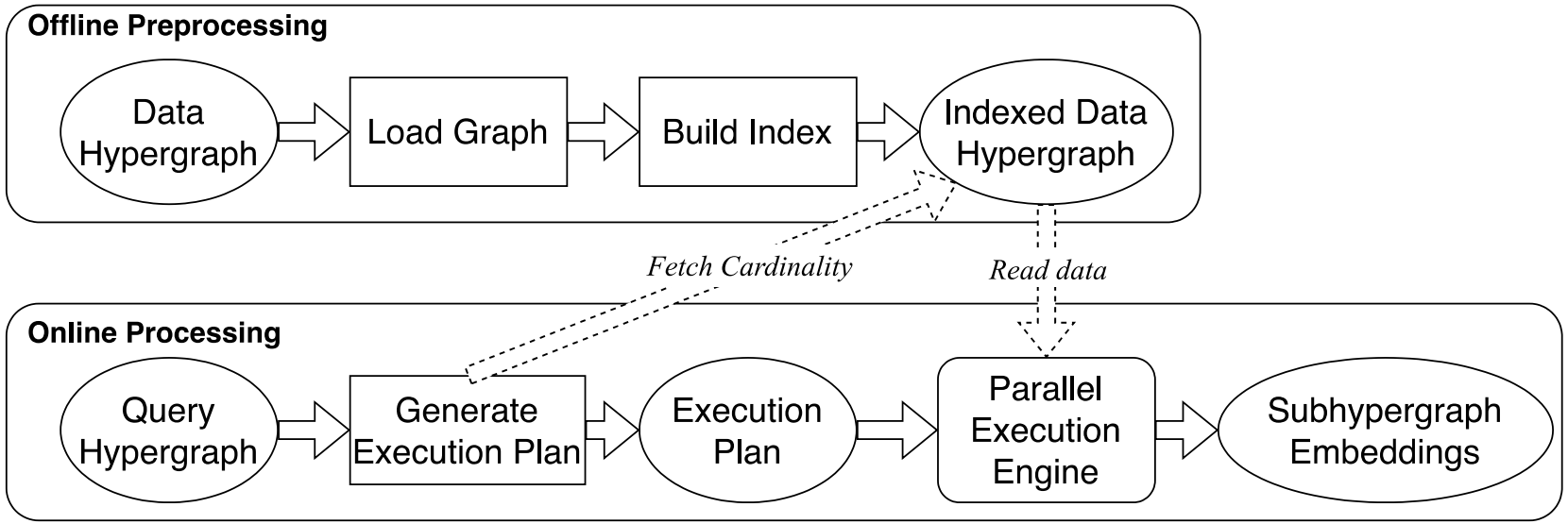
Motivations

1. The *match-by-vertex* approach in the strawman approaches generally underutilise high-order information in hypergraphs
   - hyperedges are used as a verification condition in the match-by-vertex framework, which can lead to a huge search space and large enumeration cost
2. It is difficult to compute subgraph matching on *massive hypergraphs* using sequential algorithms
   - none of the existing subhypergraph matching algorithms supports parallel execution

Contributions

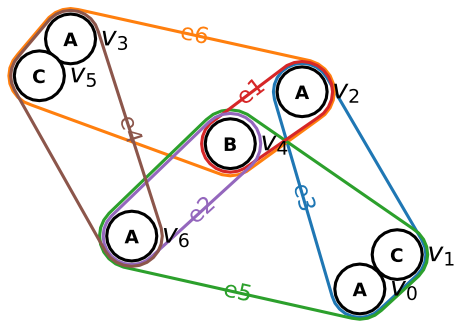
1. **A match-by-hyperedge framework**
   - Match the query by hyperedges instead of vertices
   - Use set operations to efficiently generate candidates
   - Filter out false positives with set comparison
2. **A highly optimised parallel execution engine**
   - Adopt the dataflow model for parallelisation
   - Bounded memory consumption with our task-based scheduler
   - Load balancing with dynamic work-stealing

HGMatch Overview



Hypergraph Data Layout

- Hypergraphs are stored as ***hyperedge tables*** with ***inverted hyperedge index***
  - Hyperedge Signature: a *multiset* of all vertex labels contained in a hyperedge



Hyperedge Signature

Hyperedges

Inverted Hyperedge Index

(a) Partition 1

$\mathcal{S}(e) = \{A, B\}$	
E	$e_1 = \{v_2, v_4\}$
	$e_2 = \{v_4, v_6\}$
I	$v_2 \rightarrow [e_1]$
	$v_4 \rightarrow [e_1, e_2]$
	$v_6 \rightarrow [e_2]$

(b) Partition 2

$\mathcal{S}(e) = \{A, A, C\}$	
E	$e_3 = \{v_0, v_1, v_2\}$
	$e_4 = \{v_3, v_5, v_6\}$
I	$v_0, v_1, v_2 \rightarrow [e_3]$
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(c) Partition 3

$\mathcal{S}(e) = \{A, A, B, C\}$	
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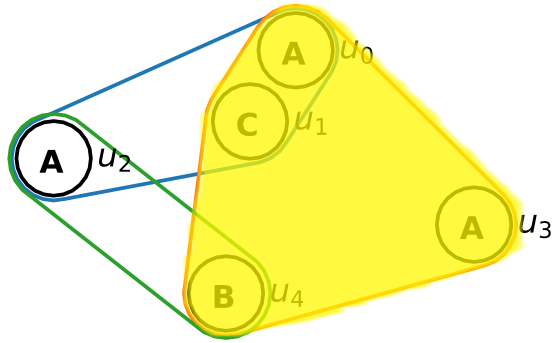
Match-by-Hyperedge Framework



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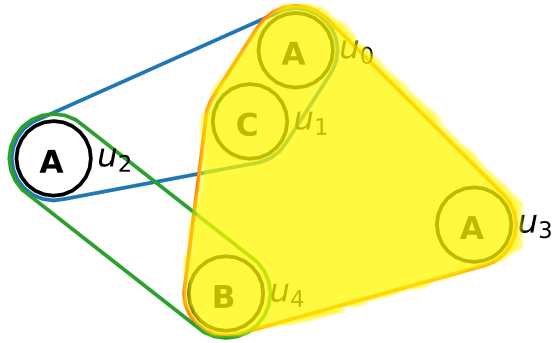

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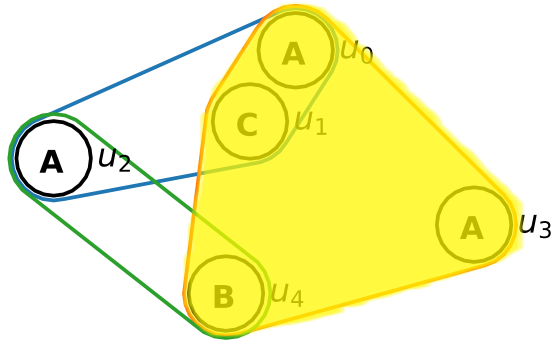

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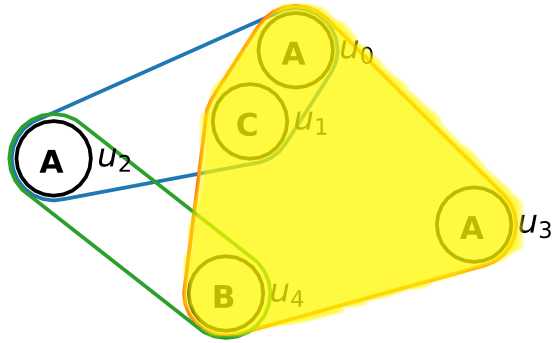
- e must be incident to $v_4 \in e_1$ and $v_0, v_1 \in e_3$

Match-by-Hyperedge Framework



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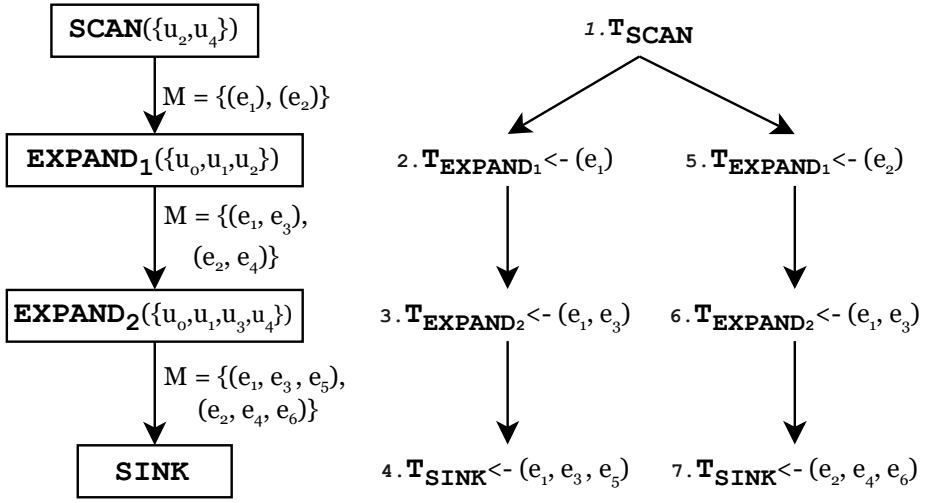
- e must have the same signature with the query hyperedge
- e must be incident to $v_4 \in e_1$ and $v_0, v_1 \in e_3$

$$\Longrightarrow C(e) = \{e_5\} \cap \{e_5\} \cap \{e_5, e_6\} = \{e_5\}$$

Parallel Execution

- Dataflow Model
  - We designed three operators: SCAN, EXPAND, SINK
- Task-based Scheduler
  - Computation are broken down into tasks and scheduled in LIFO order to bound memory
- Dynamic Work Stealing
  - Idle worker will steal tasks from others for load balancing



Example Dataflow Graph and Task Tree

Experimental Setup

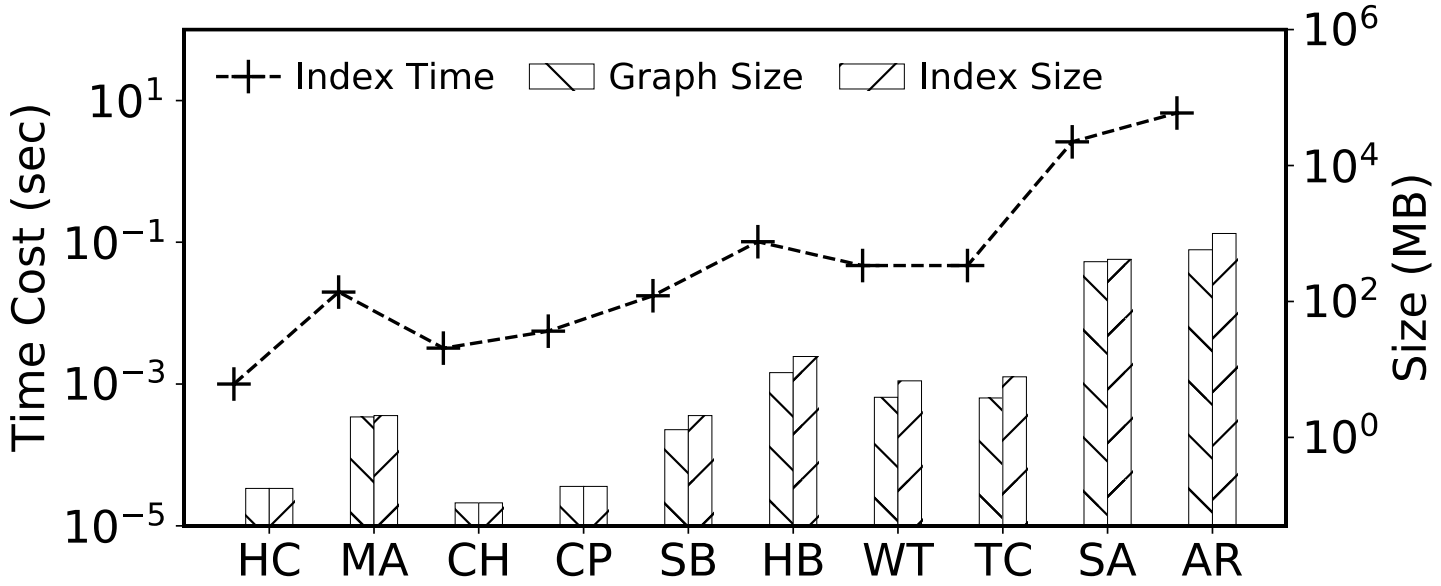
- **Hardware**: a server with two 20-core Xeon E5-2698 V4 CPU and 512G of memory
- **Baselines**: we propose a generic framework to extend existing subgraph matching algorithms to the case of hypergraphs
  - We compared the extended version of *CFL* (SIGMOD16), *DAF* (SIGMOD19), *CECI* (SIGMOD19), and *RapidMatch* (VLDB20)
- **Queries**: randomly sample subhypergraphs from the data hypergraphs with given number of hyperedges and vertices

Datasets

- **Datasets**: we use 10 real-world hypergraphs as data hypergraphs

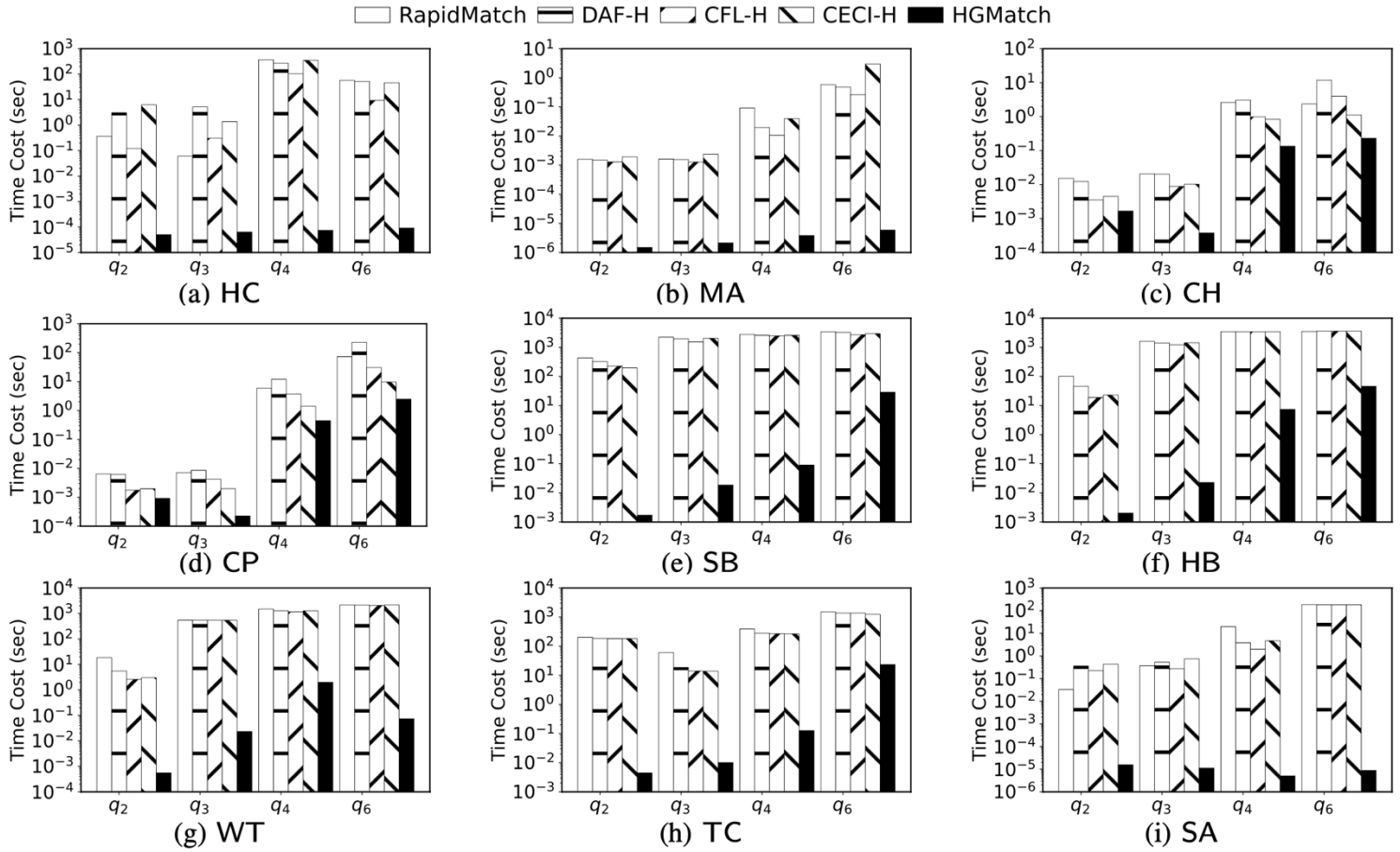
Dataset	$\|V\|$	$\|E\|$	$\|\Sigma\|$	a_{max}	$\bar{a}$	$\|Index\|$
HC	1,290	331	2	81	34.8	178KB
MA	73,851	5,444	1,456	1,784	24.2	2.1MB
CH	327	7,818	9	5	2.3	109KB
CP	242	12,704	11	5	2.4	190KB
SB	294	20,584	2	99	8.0	2.1MB
HB	1,494	52,960	2	399	20.5	15.5MB
WT	88,860	65,507	11	25	6.6	6.8MB
TC	172,738	212,483	160	85	4.1	7.8MB
SA	15,211,989	1,103,193	56,502	61,315	23.7	419.7MB
AR	2,268,264	4,239,108	29	9,350	17.1	998.6MB

Index Building



Building Time and Size of Index

Single-thread Comparisons



Execution Time for each Query Set

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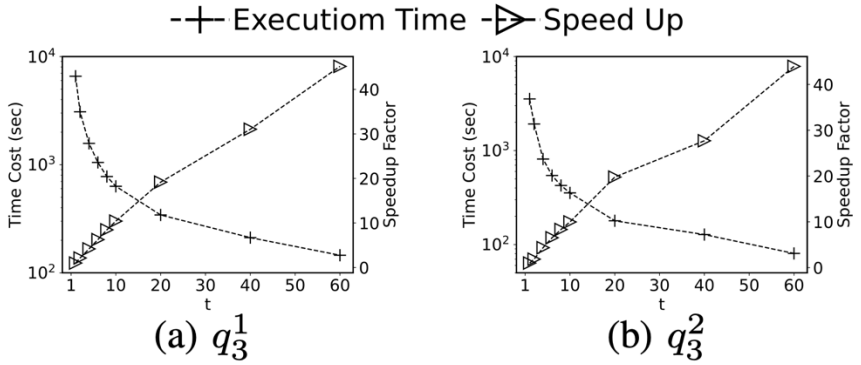
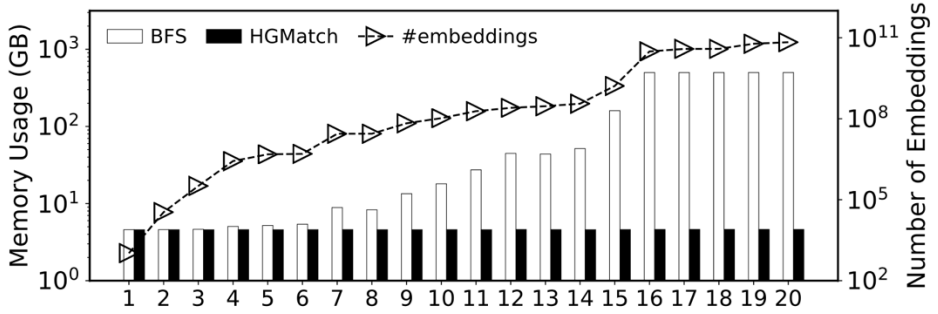
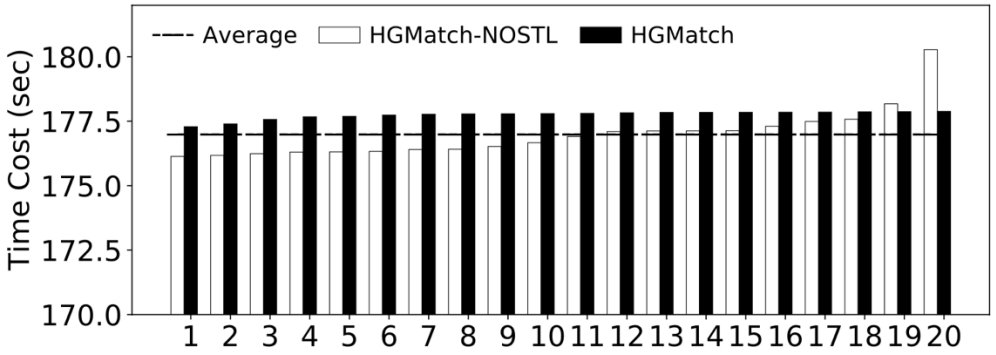
Parallel Comparisons



Vary Number of Threads



Task-based Scheduling



Work Stealing

Thank you!

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